

Working a Leslie Matrix Equation

$$\begin{matrix}
 \mathbf{L} & & \mathbf{N}_t & = & \mathbf{N}_{t+1} \\
 \begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 \\ s_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 \end{bmatrix} & \times & \begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \end{bmatrix} & = & \begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \end{bmatrix}
 \end{matrix}$$

The population vector (\mathbf{N}_t) is a column of numbers, where each number is the number of females in a particular age class at a particular time ($n_{x,t}$).

The Leslie matrix (\mathbf{L}) is a square matrix of numbers, where the top row consists of age-specific fertility rates (m_x) and the subdiagonal consists of age-specific survival rates (s_x). The rest of the numbers in the matrix are all zeroes.

For each row in the Leslie matrix (\mathbf{L}), each value gets multiplied by the corresponding age class in the current population vector (\mathbf{N}_t), and the products are added up to produce the value in the projected population vector (\mathbf{N}_{t+1}) in that row.

Projected newborns ($n_{0,t+1}$) is the total reproductive output over all age classes

$$\begin{matrix}
 \mathbf{L} & & \mathbf{N}_t & = & \mathbf{N}_{t+1} \\
 \begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 \\ s_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 \end{bmatrix} & \times & \begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \end{bmatrix} & = & \begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \end{bmatrix} \\
 \hline
 m_0^x n_{0,t} + m_1^x n_{1,t} + m_2^x n_{2,t} + m_3^x n_{3,t} + m_4^x n_{4,t} + m_5^x n_{5,t} & = & n_{0,t+1} \\
 \hline
 m_0 n_{0,t} + m_1 n_{1,t} + m_2 n_{2,t} + m_3 n_{3,t} + m_4 n_{4,t} + m_5 n_{5,t} & = & n_{0,t+1}
 \end{matrix}$$

For each row in the Leslie matrix (\mathbf{L}), each value gets multiplied by the corresponding age class in the current population vector (\mathbf{N}_t), and the products are added up to produce the value in the projected population vector (\mathbf{N}_{t+1}) in that row.

Projected 1-year olds ($n_{1,t+1}$) are current newborns that survive.

$$\begin{matrix}
 \mathbf{L} & & \mathbf{N}_t & = & \mathbf{N}_{t+1} \\
 \begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 \\ s_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 \end{bmatrix} & \times & \begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \end{bmatrix} & = & \begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \end{bmatrix} \\
 \hline
 0^x n_{0,t} + 0^x n_{1,t} + 0^x n_{2,t} + 0^x n_{3,t} + 0^x n_{4,t} + 0^x n_{5,t} & = & n_{1,t+1} \\
 \hline
 s_0 n_{0,t} & = & n_{1,t+1}
 \end{matrix}$$

For each row in the Leslie matrix (\mathbf{L}), each value gets multiplied by the corresponding age class in the current population vector (\mathbf{N}_t), and the products are added up to produce the value in the projected population vector (\mathbf{N}_{t+1}) in that row.

Projected 2-year olds ($n_{2,t+1}$) are current 1-year olds that survive.

$$\begin{matrix}
 \mathbf{L} & & \mathbf{N}_t & = & \mathbf{N}_{t+1} \\
 \begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 \\ s_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 \end{bmatrix} & \times & \begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \end{bmatrix} & = & \begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \end{bmatrix} \\
 \hline
 0^x n_{0,t} + s_1^x n_{1,t} + 0^x n_{2,t} + 0^x n_{3,t} + 0^x n_{4,t} + 0^x n_{5,t} & = & n_{2,t+1} \\
 \hline
 s_1 n_{1,t} & = & n_{2,t+1}
 \end{matrix}$$

For each row in the Leslie matrix (\mathbf{L}), each value gets multiplied by the corresponding age class in the current population vector (\mathbf{N}_t), and the products are added up to produce the value in the projected population vector (\mathbf{N}_{t+1}) in that row.

Projected 3-year olds ($n_{3,t+1}$) are current 2-year olds that survive.

$$\begin{matrix}
 \mathbf{L} & & \mathbf{N}_t & = & \mathbf{N}_{t+1} \\
 \begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 \\ s_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 \end{bmatrix} & \times & \begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \end{bmatrix} & = & \begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \end{bmatrix} \\
 \hline
 0^x n_{0,t} + 0^x n_{1,t} + s_2^x n_{2,t} + 0^x n_{3,t} + 0^x n_{4,t} + 0^x n_{5,t} & = & n_{3,t+1} \\
 \hline
 s_2 n_{2,t} & = & n_{3,t+1}
 \end{matrix}$$

And so on.....