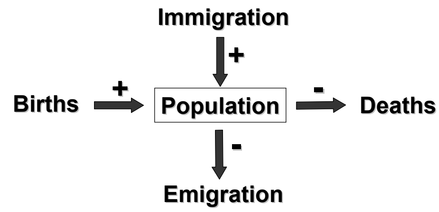
Population Growth

Population Dynamics



changes in population size that result from variations in the rates of birth, death and movement of individuals

An Old, Bad Joke

- Why are rabbits the smartest animals in the world?

- Answer: because the know how to **multiply** so well!

POPULATION GROWTH IS EXPONENTIAL: A PROCESS OF REPEATED MULTIPLICATION

First Law of Population Growth

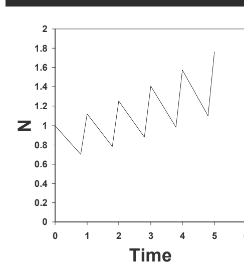
- If *per capita* growth rate is constant, a population will grow exponentially.

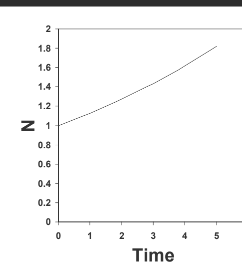
$N_{t+\Delta t} = N_t * X^{\Delta t}$

Discrete vs. Continuous Population Growth

- **Discrete**
  - Reproduction occurs in distinct pulses (usually annual)
  - Newborns cannot reproduce until next pulse
- **Continuous**
  - Reproduction occurs throughout the year or growing season
  - Newborns can reproduce in the year they are born
  - “Compounding” occurs within each time step (e.g., year)

Discrete vs. Continuous Population Growth





Discrete vs. Continuous Population Growth: Credit Cards

Mr. Nice Guy Bank:
Finance Charges (12% APR) Compounded Annually

You owe \$100 today, and don't make any payments (assume no late fees!)

In 1 year you will owe:

\$100 X 1.12 = \$112

Discrete vs. Continuous Population Growth: Credit Cards

Snitty Bank: Finance Charges (12% APR) Compounded Monthly (12%/12 months = 1%/month)

You owe \$100 today, and don't make any payments (assume no late fees!)

In 1 month you will owe:
\$100 X 1.01 = \$101

In 2 months you will owe: \$100 X 1.01 X 1.01 = \$102.01

In 1 year you will owe:
\$100 X $(1.01)^{12}$ = \$112.68

Discrete vs. Continuous Population Growth: Credit Cards

Instantaneous Bank of America:
Finance Charges (12%) Compounded Instantaneously

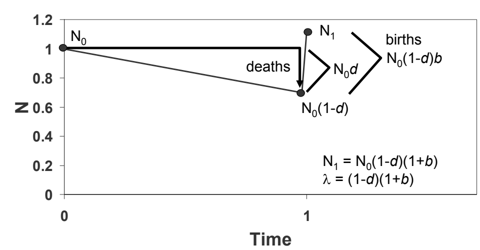
You owe \$100 today, and don't make any payments (assume no late fees!)

In 1 year you will owe:

\$100 X $e^{0.12}$ = \$112.75

e = 2.71828183… (root of the natural logarithm)

Describing Population Growth

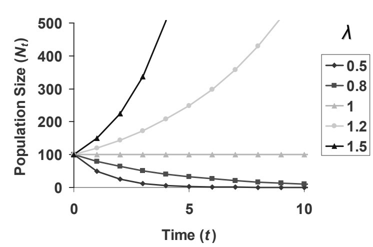


Discrete vs. Continuous Exponential Population Growth

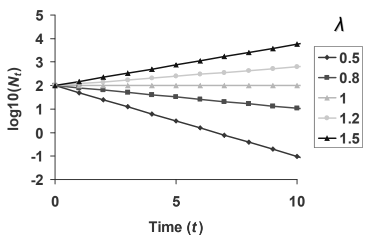
- **Discrete**
  $\lambda = (1 - d)(1 + b)$
  Total Growth Rate: $N_{t+1} - N_t = (1-\lambda)N_t$
  Solution: $N_t = N_0 * \lambda^t$
- **Continuous**
  $r = b - d$
  Total Growth Rate: $dN/dt = rN$
  Solution: $N_t = N_0 * e^{rt}$
  (same as discrete, but with $\lambda = e^r$)

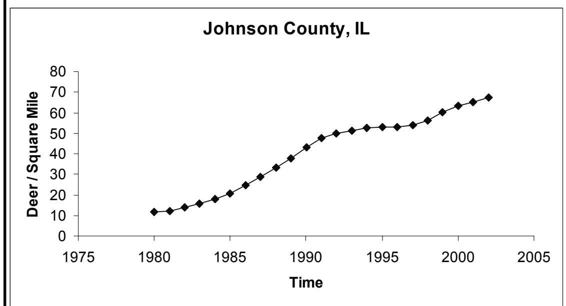
Either way, doubling time = log(2)/log(λ)

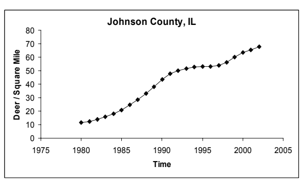
Exponential Growth

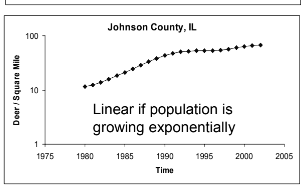


Exponential Growth









Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3			
1991	47.6			
1992	50.0			
1993	51.4			
1994	52.5			
1995	53.0			
1996	52.9			
1997	53.7			
1998	56.3			
1999	60.1			
2000	63.4			

Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	(47.6-43.3)/1		
1991	47.6			
1992	50.0			
1993	51.4			
1994	52.5			
1995	53.0			
1996	52.9			
1997	53.7			
1998	56.3			
1999	60.1			
2000	63.4			

Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	4.3		
1991	47.6	2.4		
1992	50.0	1.4		
1993	51.4	1.1		
1994	52.5	0.5		
1995	53.0	-0.1		
1996	52.9	0.8		
1997	53.7	2.6		
1998	56.3	3.8		
1999	60.1	3.3		
2000	63.4			

Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	4.3	4.3/43.3	
1991	47.6	2.4		
1992	50.0	1.4		
1993	51.4	1.1		
1994	52.5	0.5		
1995	53.0	-0.1		
1996	52.9	0.8		
1997	53.7	2.6		
1998	56.3	3.8		
1999	60.1	3.3		
2000	63.4			

Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	4.3	0.098	
1991	47.6	2.4	0.050	
1992	50.0	1.4	0.029	
1993	51.4	1.1	0.022	
1994	52.5	0.5	0.009	
1995	53.0	-0.1	-0.002	
1996	52.9	0.8	0.016	
1997	53.7	2.6	0.047	
1998	56.3	3.8	0.068	
1999	60.1	3.3	0.055	
2000	63.4			

Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	4.3	0.098	47.6/43.3
1991	47.6	2.4	0.050	
1992	50.0	1.4	0.029	
1993	51.4	1.1	0.022	
1994	52.5	0.5	0.009	
1995	53.0	-0.1	-0.002	
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Year	Deer/mi^2	Total Growth Rate (N_{t+1}-N_t)	Per capita Growth	λ (N_{t+1}/N_t)
1990	43.3	4.3	0.098	1.098
1991	47.6	2.4	0.050	1.050
1992	50.0	1.4	0.029	1.029
1993	51.4	1.1	0.022	1.022
1994	52.5	0.5	0.009	1.009
1995	53.0	-0.1	-0.002	0.998
1996	52.9	0.8	0.016	1.016
1997	53.7	2.6	0.047	1.047
1998	56.3	3.8	0.068	1.068
1999	60.1	3.3	0.055	1.055
2000	63.4			

Population Growth With Age Structure

By convention, we'll just look at females

Population size (# females) is the sum of females in all age classes

$N_t = n_{0,t} + n_{1,t} + n_{2,t} + n_{3,t} + ...$

Except for newborns, individuals in each age class in year *t* are survivors of the previous age class the previous year:

$n_{1,t+1} = n_{0,t} \times s_0$

$n_{2,t+1} = n_{1,t} \times s_1$ ... and so on...

Population Growth With Age Structure

Animals in age class 0 are products of reproduction within the last time step:

$n_{0,t+1} = n_{0,t}m_0\ n_{1,t}m_1 + n_{2,t}m_2 + n_{3,t}m_3 + ...$

[m_x = #females born per female in age class x]

Population Growth With Age Structure

So, putting it all together, if there are 7 age classes (0-6):

$n_{0,t} = n_{0,t-1}m_0 + n_{1,t-1}m_1 + n_{2,t}m_2 + n_{3,t}m_3 + n_{4,t}m_4 + n_{5,t}m_5 + n_{6,t}m_6$
$n_{1,t} = n_{0,t-1}s_0$
$n_{2,t} = n_{1,t-1}s_1$
$n_{3,t} = n_{2,t-1}s_2$
$n_{4,t} = n_{3,t-1}s_3$
$n_{5,t} = n_{4,t-1}s_4$
$n_{6,t} = n_{5,t-1}s_5$
$N_t = n_{0,t} + n_{1,t} + n_{2,t} + n_{3,t} + n_{4,t} + n_{5,t} + n_{6,t}$

THAT'S A LOT OF WORK JUST TO WRITE!
Maybe there's an easier way...

Population Growth With Age Structure: MATRIX MODEL

$$\mathbf{L} \times \mathbf{N}_t = \mathbf{N}_{t+1}$$

$$\underset{\mathbf{L}}{\begin{bmatrix} m_0 & m_1 & m_2 & m_3 & m_4 & m_5 & m_6 \\ s_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_5 & 0 \end{bmatrix}} \times \underset{\mathbf{N}_t}{\begin{bmatrix} n_{0,t} \\ n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ n_{4,t} \\ n_{5,t} \\ n_{6,t} \end{bmatrix}} = \underset{\mathbf{N}_{t+1}}{\begin{bmatrix} n_{0,t+1} \\ n_{1,t+1} \\ n_{2,t+1} \\ n_{3,t+1} \\ n_{4,t+1} \\ n_{5,t+1} \\ n_{6,t+1} \end{bmatrix}}$$

Leslie Projection Matrix — *population vector*

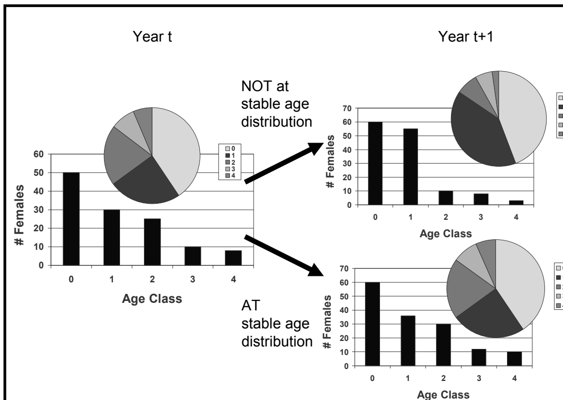
Population Growth With Age Structure: MATRIX MODEL

$$\mathbf{L} \times \mathbf{N}_t = \mathbf{N}_{t+1}$$

As this process is repeated, the population converges on a <u>**stable age distribution**</u> (SAD)

as t gets large, $n_{x,t}/N_t$ approaches a constant value (different for each age class, but constant over time)

growth is truly exponential after SAD is achieved



Population Growth With Age Structure: MATRIX MODEL

$$\mathbf{L} \times \mathbf{N}_t = \mathbf{N}_{t+1}$$

Why are you doing this to us, Evil Dr. Schauber? What's the point of making these horrible matrices?

Using linear algebra ("matrix math"), **L** can give us:
- the **stable age distribution** (= dominant right eigenvector)
- the **asymptotic per capita growth rate** (= dominant eigenvalue)
- the **reproductive value** of each age class (= dominant left eigenvector)
    - (r.v. is the contribution of each individual in an age class to overall pop. growth rate)

Population Growth With Age Structure: MATRIX MODEL

How can matrix models be useful for management?

Things to Remember

- Exponential growth: discrete vs. continuous time
  - both are linear on a logarithmic scale
- Age structure: Matrix models
  - what are **N** and **L**?
  - what are their properties?
  - what is stable age distribution?
  - advantages of matrix approach
  - utility for management